90 Minutes—No Calculator

Note: In this examination, ln x denotes the natural logarithm of x (that is, logarithm to the base e).

1. $\int \left(x^3 - 3x\right) dx =$

(A) $3x^2 - 3 + C$ (B) $4x^4 - 6x^2 + C$ (C) $\frac{x^4}{3} - 3x^2 + C$

(D) $\frac{x^4}{4} - 3x + C$ (E) $\frac{x^4}{4} - \frac{3x^2}{2} + C$

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2. If $f(x) = x^3 + 3x^2 + 4x + 5$ and $g(x) = 5$, then $g\left(f(x)\right) =$

(A) $5x^2 + 15x + 25$ (B) $5x^3 + 15x^2 + 20x + 25$ (C) 1125

(D) 225 (E) 5

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3. The slope of the line tangent to the graph of $y = \ln\left(x^2\right)$ at $x = e^2$ is

(A) $\frac{1}{e^2}$ (B) $\frac{2}{e^2}$ (C) $\frac{4}{e^2}$ (D) $\frac{1}{e^4}$ (E) $\frac{4}{e^4}$

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4. If $f(x) = x + \sin x$, then $f'(x) =$

(A) $1 + \cos x$ (B) $1 - \cos x$ (C) $\cos x$

(D) $\sin x - x\cos x$ (E) $\sin x + x\cos x$

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5. If $f(x) = e^x$, which of the following lines is an asymptote to the graph of f?

(A) $y = 0$ (B) $x = 0$ (C) $y = x$ (D) $y = -x$ (E) $y = 1$

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6. If $f(x) = \frac{x-1}{x+1}$ for all $x \neq -1$, then $f'(1) =$

(A) -1 (B) $-\frac{1}{2}$ (C) 0 (D) $\frac{1}{2}$ (E) 1

7. Which of the following equations has a graph that is symmetric with respect to the origin?

(A) $y = \frac{x+1}{x}$ (B) $y = -x^5 + 3x$ (C) $y = x^4 - 2x^2 + 6$

(D) $y = (x-1)^3 + 1$ (E) $y = \left(x^2+1\right)^2 - 1$

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8. A particle moves in a straight line with velocity $v(t) = t^2$. How far does the particle move between times $t = 1$ and $t = 2$?

(A) $\frac{1}{3}$ (B) $\frac{7}{3}$ (C) 3 (D) 7 (E) 8

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9. If $y = \cos^2 3x$, then $\frac{dy}{dx} =$

(A) $-6\sin 3x \cos 3x$ (B) $-2\cos 3x$ (C) $2\cos 3x$

(D) $6\cos 3x$ (E) $2\sin 3x \cos 3x$

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10. The *derivative* of $f(x) = \frac{x^4}{3} - \frac{x^5}{5}$ attains its maximum value at $x =$

(A) –1 (B) 0 (C) 1 (D) $\frac{4}{3}$ (E) $\frac{5}{3}$

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11. If the line $3x - 4y = 0$ is tangent in the first quadrant to the curve $y = x^3 + k$, then k is

(A) $\frac{1}{2}$ (B) $\frac{1}{4}$ (C) 0 (D) $-\frac{1}{8}$ (E) $-\frac{1}{2}$

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12. If $f(x) = 2x^3 + Ax^2 + Bx - 5$ and if $f(2) = 3$ and $f(-2) = -37$, what is the value of $A + B$?

(A) –6 (B) –3 (C) –1 (D) 2

(E) It cannot be determined from the information given.

13. The acceleration α of a body moving in a straight line is given in terms of time t by $\alpha = 8 - 6t$. If the velocity of the body is 25 at $t = 1$ and if $s(t)$ is the distance of the body from the origin at time t, what is $s(4) - s(2)$?

(A) 20 (B) 24 (C) 28 (D) 32 (E) 42

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14. If $f(x) = x^{\frac{1}{3}}(x-2)^{\frac{2}{3}}$ for all x, then the domain of f' is

(A) $\{x \mid x \neq 0\}$ (B) $\{x \mid x > 0\}$ (C) $\{x \mid 0 \leq x \leq 2\}$

(D) $\{x \mid x \neq 0 \text{ and } x \neq 2\}$ (E) $\{x \mid x \text{ is a real number}\}$

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15. The area of the region bounded by the lines $x = 0$, $x = 2$, and $y = 0$ and the curve $y = e^{\frac{x}{2}}$ is

(A) $\frac{e-1}{2}$ (B) $e-1$ (C) $2(e-1)$ (D) $2e-1$ (E) $2e$

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16. The number of bacteria in a culture is growing at a rate of $3000e^{\frac{2t}{5}}$ per unit of time t. At $t = 0$, the number of bacteria present was 7,500. Find the number present at $t = 5$.

(A) $1,200e^2$ (B) $3,000e^2$ (C) $7,500e^2$ (D) $7,500e^5$ (E) $\frac{15,000}{7}e^7$

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17. What is the area of the region completely bounded by the curve $y = -x^2 + x + 6$ and the line $y = 4$?

(A) $\frac{3}{2}$ (B) $\frac{7}{3}$ (C) $\frac{9}{2}$ (D) $\frac{31}{6}$ (E) $\frac{33}{2}$

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18. $\frac{d}{dx}(\arcsin 2x) =$

(A) $\frac{-1}{2\sqrt{1-4x^2}}$ (B) $\frac{-2}{\sqrt{4x^2-1}}$ (C) $\frac{1}{2\sqrt{1-4x^2}}$

(D) $\frac{2}{\sqrt{1-4x^2}}$ (E) $\frac{2}{\sqrt{4x^2-1}}$

19. Suppose that f is a function that is defined for all real numbers. Which of the following conditions assures that f has an inverse function?

(A) The function f is periodic.

(B) The graph of f is symmetric with respect to the y-axis.

(C) The graph of f is concave up.

(D) The function f is a strictly increasing function.

(E) The function f is continuous.

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20. If F and f are continuous functions such that $F'(x) = f(x)$ for all x, then $\int_a^b f(x)\,dx$ is

(A) $F'(a) - F'(b)$

(B) $F'(b) - F'(a)$

(C) $F(a) - F(b)$

(D) $F(b) - F(a)$

(E) none of the above

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21. $\int_0^1 (x+1)e^{x^2+2x}\,dx =$

(A) $\frac{e^3}{2}$ (B) $\frac{e^3-1}{2}$ (C) $\frac{e^4-e}{2}$ (D) e^3-1 (E) e^4-e

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22. Given the function defined by $f(x) = 3x^5 - 20x^3$, find all values of x for which the graph of f is concave up.

(A) $x > 0$

(B) $-\sqrt{2} < x < 0$ or $x > \sqrt{2}$

(C) $-2 < x < 0$ or $x > 2$

(D) $x > \sqrt{2}$

(E) $-2 < x < 2$

23. $\lim\limits_{h\to 0} \frac{1}{h}\ln\left(\frac{2+h}{2}\right)$ is

(A) e^2 (B) 1 (C) $\frac{1}{2}$ (D) 0 (E) nonexistent

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24. Let $f(x)=\cos(\arctan x)$. What is the range of f?

(A) $\left\{x \,\middle|\, -\frac{\pi}{2}<x<\frac{\pi}{2}\right\}$ (B) $\{x \mid 0<x\le 1\}$ (C) $\{x \mid 0\le x\le 1\}$

(D) $\{x \mid -1<x<1\}$ (E) $\{x \mid -1\le x\le 1\}$

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25. $\int_0^{\pi/4} \tan^2 x\,dx =$

(A) $\frac{\pi}{4}-1$ (B) $1-\frac{\pi}{4}$ (C) $\frac{1}{3}$ (D) $\sqrt{2}-1$ (E) $\frac{\pi}{4}+1$

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26. The radius r of a sphere is increasing at the uniform rate of 0.3 inches per second. At the instant when the surface area S becomes 100π square inches, what is the rate of increase, in cubic inches per second, in the volume V? $\left(S=4\pi r^2 \text{ and } V=\frac{4}{3}\pi r^3\right)$

(A) 10π (B) 12π (C) 22.5π (D) 25π (E) 30π

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27. $\int_0^{1/2} \frac{2x}{\sqrt{1-x^2}}\,dx=$

(A) $1-\frac{\sqrt{3}}{2}$ (B) $\frac{1}{2}\ln\frac{3}{4}$ (C) $\frac{\pi}{6}$ (D) $\frac{\pi}{6}-1$ (E) $2-\sqrt{3}$

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28. A point moves in a straight line so that its distance at time t from a fixed point of the line is $8t-3t^2$. What is the *total* distance covered by the point between $t=1$ and $t=2$?

(A) 1 (B) $\frac{4}{3}$ (C) $\frac{5}{3}$ (D) 2 (E) 5

29. Let $f(x) = \left| \sin x - \frac{1}{2} \right|$. The maximum value attained by f is

(A) $\frac{1}{2}$ (B) 1 (C) $\frac{3}{2}$ (D) $\frac{\pi}{2}$ (E) $\frac{3\pi}{2}$

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30. $\int_1^2 \frac{x-4}{x^2}\,dx =$

(A) $-\frac{1}{2}$ (B) $\ln 2 - 2$ (C) $\ln 2$ (D) 2 (E) $\ln 2 + 2$

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31. If $\log_a\left(2^a\right) = \frac{a}{4}$, then $a =$

(A) 2 (B) 4 (C) 8 (D) 16 (E) 32

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32. $\int \frac{5}{1+x^2}\,dx =$

(A) $\frac{-10x}{\left(1+x^2\right)^2} + C$ (B) $\frac{5}{2x}\ln\left(1+x^2\right) + C$ (C) $5x - \frac{5}{x} + C$

(D) $5\arctan x + C$ (E) $5\ln\left(1+x^2\right) + C$

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33. Suppose that f is an odd function; i.e., $f(-x) = -f(x)$ for all x. Suppose that $f'\left(x_0\right)$ exists. Which of the following must necessarily be equal to $f'\left(-x_0\right)$?

(A) $f'\left(x_0\right)$

(B) $-f'\left(x_0\right)$

(C) $\frac{1}{f'\left(x_0\right)}$

(D) $\frac{-1}{f'\left(x_0\right)}$

(E) None of the above

34. The average value of $\sqrt{x}$ over the interval $0 \le x \le 2$ is

(A) $\frac{1}{3}\sqrt{2}$ (B) $\frac{1}{2}\sqrt{2}$ (C) $\frac{2}{3}\sqrt{2}$ (D) 1 (E) $\frac{4}{3}\sqrt{2}$

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35. The region in the first quadrant bounded by the graph of $y = \sec x$, $x = \frac{\pi}{4}$, and the axes is rotated about the x-axis. What is the volume of the solid generated?

(A) $\frac{\pi^2}{4}$ (B) $\pi - 1$ (C) π (D) 2π (E) $\frac{8\pi}{3}$

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36. If $y = e^{nx}$, then $\frac{d^n y}{dx^n} =$

(A) $n^n e^{nx}$ (B) $n!e^{nx}$ (C) $n e^{nx}$ (D) $n^n e^{x}$ (E) $n!e^{x}$

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37. If $\frac{dy}{dx} = 4y$ and if $y = 4$ when $x = 0$, then $y =$

(A) $4e^{4x}$ (B) e^{4x} (C) $3 + e^{4x}$ (D) $4 + e^{4x}$ (E) $2x^2 + 4$

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38. If $\int_1^2 f(x-c)\,dx = 5$ where c is a constant, then $\int_{1-c}^{2-c} f(x)\,dx =$

(A) $5 + c$ (B) 5 (C) $5 - c$ (D) $c - 5$ (E) –5

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39. The point on the curve $2y = x^2$ nearest to $(4,1)$ is

(A) $(0,0)$ (B) $(2,2)$ (C) $(\sqrt{2},1)$ (D) $(2\sqrt{2},4)$ (E) $(4,8)$

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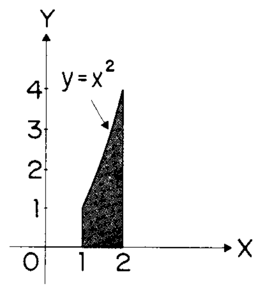
40. If $\tan(xy) = x$, then $\frac{dy}{dx} =$

(A) $\frac{1 - y\tan(xy)\sec(xy)}{x\tan(xy)\sec(xy)}$ (B) $\frac{\sec^2(xy) - y}{x}$ (C) $\cos^2(xy)$

(D) $\frac{\cos^2(xy)}{x}$ (E) $\frac{\cos^2(xy) - y}{x}$

41. Given $f(x)=\begin{cases} x+1 & \text{for } x<0, \\ \cos \pi x & \text{for } x\geq 0, \end{cases}$ $\int_{-1}^{1} f(x)\,dx =$

(A) $\frac{1}{2}+\frac{1}{\pi}$ (B) $-\frac{1}{2}$ (C) $\frac{1}{2}-\frac{1}{\pi}$ (D) $\frac{1}{2}$ (E) $-\frac{1}{2}+\pi$



42. Calculate the approximate area of the shaded region in the figure by the trapezoidal rule, using divisions at $x=\frac{4}{3}$ and $x=\frac{5}{3}$.

(A) $\frac{50}{27}$ (B) $\frac{251}{108}$ (C) $\frac{7}{3}$ (D) $\frac{127}{54}$ (E) $\frac{77}{27}$

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43. If the solutions of $f(x)=0$ are -1 and 2, then the solutions of $f\left(\frac{x}{2}\right)=0$ are

(A) -1 and 2

(B) $-\frac{1}{2}$ and $\frac{5}{2}$

(C) $-\frac{3}{2}$ and $\frac{3}{2}$

(D) $-\frac{1}{2}$ and 1

(E) -2 and 4

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44. For small values of h, the function $\sqrt[4]{16+h}$ is best approximated by which of the following?

(A) $4+\frac{h}{32}$

(B) $2+\frac{h}{32}$

(C) $\frac{h}{32}$

(D) $4-\frac{h}{32}$

(E) $2-\frac{h}{32}$

45. If f is a continuous function on $[a,b]$, which of the following is necessarily true?

(A) f' exists on (a,b).

(B) If $f(x_0)$ is a maximum of f, then $f'(x_0)=0$.

(C) $\lim_{x\to x_0} f(x) = f\left(\lim_{x\to x_0} x\right)$ for $x_0 \in (a,b)$

(D) $f'(x)=0$ for some $x \in [a,b]$

(E) The graph of f' is a straight line.

1973 Answer Key

1973 AB

1. E
2. E
3. B
4. A
5. A
6. D
7. B
8. B
9. A
10. C
11. B
12. C
13. D
14. D
15. C
16. C
17. C
18. D
19. D
20. D
21. B
22. B
23. C
24. B
25. B
26. E
27. E
28. C
29. C
30. B
31. D
32. D
33. A
34. C
35. C
36. A
37. A
38. B
39. B
40. E
41. D
42. D
43. E
44. B
45. C

1973 BC

1. A
2. D
3. A
4. C
5. B
6. D
7. D
8. B
9. A
10. A
11. E
12. D
13. D
14. A
15. C
16. A
17. C
18. D
19. D
20. E
21. B
22. C
23. C
24. A
25. B
26. D
27. E
28. C
29. A
30. B
31. E
32. C
33. A
34. C
35. C
36. E
37. E
38. B
39. D
40. C
41. D
42. D
43. E
44. A
45. E

1. E $\int (x^3 - 3x)\,dx = \frac{1}{4}x^4 - \frac{3}{2}x^2 + C$

2. E $g(x) = 5 \Rightarrow g\left(f(x)\right) = 5$

3. B $y = \ln x^2;\ y' = \frac{2x}{x^2} = \frac{2}{x}$. At $x = e^2$, $y' = \frac{2}{e^2}$.

4. A $f(x) = x + \sin x;\quad f'(x) = 1 - \cos x$

5. A $\lim\limits_{x\to-\infty} e^x = 0 \Rightarrow y = 0$ is a horizontal asymptote

6. D $f'(x) = \frac{(1)(x+1)-(x-1)(1)}{(x+1)^2},\ f'(1) = \frac{2}{4} = \frac{1}{2}$

7. B Replace x with $(-x)$ and see if the result is the opposite of the original. This is true for B. $-(-x)^5 + 3(-x) = x^5 - 3x = -(-x^5 + 3x)$.

8. B Distance $= \int_1^2 \left| t^2 \right| dx = \int_1^2 t^2\,dt = \frac{1}{3}t^3 \Big|_1^2 = \frac{1}{3}(2^3 - 1^3) = \frac{7}{3}$

9. A $y' = 2\cos 3x \cdot \frac{d}{dx}(\cos 3x) = 2\cos 3x \cdot (-\sin 3x) \cdot \frac{d}{dx}(3x) = 2\cos 3x \cdot (-\sin 3x) \cdot (3)$
$y' = -6\sin 3x \cos 3x$

10. C $f(x) = \frac{x^4}{3} - \frac{x^5}{5};\ f'(x) = \frac{4x^3}{3} - x^4;\ f''(x) = 4x^2 - 4x^3 = 4x^2(1-x)$
$f'' > 0$ for $x < 1$ and $f'' < 0$ for $x > 1 \Rightarrow f'$ has its maximum at $x = 1$.

11. B Curve and line have the same slope when $3x^2 = \frac{3}{4} \Rightarrow x = \frac{1}{2}$. Using the line, the point of tangency is $\left(\frac{1}{2}, \frac{3}{8}\right)$. Since the point is also on the curve, $\frac{3}{8} = \left(\frac{1}{2}\right)^3 + k \Rightarrow k = \frac{1}{4}$.

12. C Substitute the points into the equation and solve the resulting linear system.

$3 = 16 + 4A + 2B - 5$ and $-37 = -16 + 4A - 2B - 5$; $A = -3$, $B = 2 \Rightarrow A + B = -1$.

13. D $v(t) = 8t - 3t^2 + C$ and $v(1) = 25 \Rightarrow C = 20$ so $v(t) = 8t - 3t^2 + 20$.
$s(4) - s(2) = \int_2^4 v(t)\,dt = (4t^2 - t^3 + 20t)\Big|_2^4 = 32$

14. D $f(x) = x^{1/3}(x-2)^{2/3}$
$f'(x) = x^{1/3} \cdot \frac{2}{3}(x-2)^{-1/3} + (x-2)^{2/3} \cdot \frac{1}{3}x^{-2/3} = \frac{1}{3}x^{-2/3}(x-2)^{-1/3}(3x-2)$
f' is not defined at $x = 0$ and at $x = 2$.

15. C Area $= \int_0^2 e^{\frac{x}{2}}\,dx = 2e^{\frac{x}{2}}\Big|_0^2 = 2(e-1)$

16. C $\frac{dN}{dt} = 3000e^{\frac{2}{5}t}$, $N = 7500e^{\frac{2}{5}t} + C$ and $N(0) = 7500 \Rightarrow C = 0$
$N = 7500e^{\frac{2}{5}t}$, $N(5) = 7500e^2$

17. C Determine where the curves intersect. $-x^2 + x + 6 = 4 \Rightarrow x^2 - x - 2 = 0$
$(x-2)(x+1) = 0 \Rightarrow x = -1,\ x = 2$. Between these two x values the parabola lies above the line $y = 4$.
Area $= \int_{-1}^2 \left((-x^2 + x + 6) - 4\right)dx = \left(-\frac{1}{3}x^3 + \frac{1}{2}x^2 + 2x\right)\Big|_{-1}^2 = \frac{9}{2}$

18. D $\frac{d}{dx}(\arcsin 2x) = \frac{1}{\sqrt{1-(2x)^2}} \cdot \frac{d}{dx}(2x) = \frac{2}{\sqrt{1-(2x)^2}} = \frac{2}{\sqrt{1-4x^2}}$

19. D If f is strictly increasing then it must be one to one and therefore have an inverse.

20. D By the Fundamental Theorem of Calculus, $\int_a^b f(x)\,dx = F(b) - F(a)$ where $F'(x) = f(x)$.

21. B $\int_0^1 (x+1)e^{x^2+2x}\,dx = \frac{1}{2}\int_0^1 e^{x^2+2x}\left((2x+2)\,dx\right) = \frac{1}{2}\left(e^{x^2+2x}\right)\Big|_0^1 = \frac{1}{2}\left(e^3 - e^0\right) = \frac{e^3-1}{2}$

22. B $f(x) = 3x^5 - 20x^3$; $f'(x) = 15x^4 - 60x^2$; $f''(x) = 60x^3 - 120x = 60x\left(x^2 - 2\right)$
The graph of f is concave up where $f'' > 0$: $f'' > 0$ for $x > \sqrt{2}$ and for $-\sqrt{2} < x < 0$.

23. C $\lim\limits_{h\to 0}\dfrac{\ln(2+h)-\ln 2}{h}=f'(2)$ where $f(x)=\ln x$; $f'(x)=\dfrac{1}{x}\Rightarrow f'(2)=\dfrac{1}{2}$

24. B $f(x)=\cos(\arctan x)$; $-\dfrac{\pi}{2}<\arctan x<\dfrac{\pi}{2}$ and the cosine in this domain takes on all values in the interval $(0,1]$.

25. B $\int_0^{\frac{\pi}{4}}\tan^2 x\,dx=\int_0^{\frac{\pi}{4}}(\sec^2 x-1)\,dx=(\tan x-x)\Big|_0^{\pi/4}=1-\dfrac{\pi}{4}$

26. E $\dfrac{dV}{dt}=4\pi r^2\cdot\dfrac{dr}{dt}=S\cdot\dfrac{dr}{dt}=100\pi(0.3)=30\pi$

27. E $\int_0^{1/2}\dfrac{2x}{\sqrt{1-x^2}}\,dx=-\int_0^{1/2}\left(1-x^2\right)^{-\frac{1}{2}}(-2x\,dx)=-2\left(1-x^2\right)^{\frac{1}{2}}\Big|_0^{\frac{1}{2}}=2-\sqrt{3}$

28. C $v(t)=8-6t$ changes sign at $t=\dfrac{4}{3}$. Distance $=\left|x(1)-x\left(\dfrac{4}{3}\right)\right|+\left|x(2)-x\left(\dfrac{4}{3}\right)\right|=\dfrac{5}{3}$.

Alternative Solution: Distance $=\int_1^2\left|v(t)\right|dt=\int_1^2\left|8-6t\right|dt=\dfrac{5}{3}$

29. C $-1\le\sin x\le 1\Rightarrow-\dfrac{3}{2}\le\sin x-\dfrac{1}{2}\le\dfrac{1}{2}$; The maximum for $\left|\sin x-\dfrac{1}{2}\right|$ is $\dfrac{3}{2}$.

30. B $\int_1^2\dfrac{x-4}{x^2}\,dx=\int_1^2\left(\dfrac{1}{x}-4x^{-2}\right)dx=\left(\ln x+\dfrac{4}{x}\right)\Big|_1^2=(\ln 2+2)-(\ln 1+4)=\ln 2-2$

31. D $\log_a\left(2^a\right)=\dfrac{a}{4}\Rightarrow\log_a 2=\dfrac{1}{4}\Rightarrow 2=a^{\frac{1}{4}}$; $a=16$

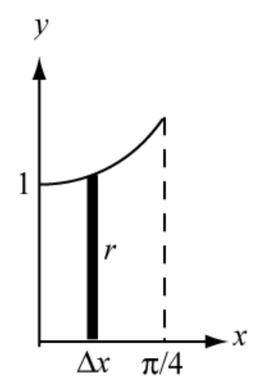
32. D $\int\dfrac{5}{1+x^2}\,dx=5\int\dfrac{1}{1+x^2}\,dx=5\tan^{-1}(x)+C$

33. A $f(-x)=-f(x)\Rightarrow f'(-x)\cdot(-1)=-f'(x)\Rightarrow f'(-x)=-f'(x)$ thus $f'(-x_0)=-f'(x_0)$.

34. C $\frac{1}{2}\int_0^2 \sqrt{x}\,dx = \frac{1}{2}\cdot\frac{2}{3}x^{\frac{3}{2}}\Big|_0^2 = \frac{1}{3}\cdot 2^{\frac{3}{2}} = \frac{2}{3}\sqrt{2}$

35. C Washers: $\sum \pi r^2 \Delta x$ where $r = y = \sec x$.

Volume $= \pi\int_0^{\frac{\pi}{4}} \sec^2 x\,dx = \pi \tan x\Big|_0^{\pi/4} = \pi(\tan\frac{\pi}{4} - \tan 0) = \pi$



36. A $y = e^{nx}$, $y' = ne^{nx}$, $y'' = n^2e^{nx}, \cdots, y^{(n)} = n^n e^{nx}$

37. A $\frac{dy}{dx} = 4y$, $y(0) = 4$. This is exponential growth. The general solution is $y = Ce^{4x}$. Since $y(0) = 4$, $C = 4$ and so the solution is $y = 4e^{4x}$.

38. B Let $z = x - c$. Then $5 = \int_1^2 f\left(x-c\right)dx = \int_{1-c}^{2-c} f\left(z\right)dz$

39. B Use the distance formula to determine the distance, L, from any point $(x, y) = (x, \frac{1}{2}x^2)$ on the curve to the point $(4,1)$. The distance L satisfies the equation $L^2 = \left(x-4\right)^2 + \left(\frac{1}{2}x^2 - 1\right)$.
Determine where L is a maximum by examining critical points. Differentiating with respect to x, $2L\cdot\frac{dL}{dx} = 2(x-4) + 2\left(\frac{1}{2}x^2 - 1\right)x = x^3 - 8$. $\frac{dL}{dx}$ changes sign from positive to negative at $x = 2$ only. The point on the curve has coordinates $(2,2)$.

40. E $\sec^2(xy)\cdot(xy' + y) = 1$, $xy'\sec^2(xy) + y\sec^2(xy) = 1$, $y' = \frac{1 - y\sec^2(xy)}{x\sec^2(xy)} = \frac{\cos^2(xy) - y}{x}$

41. D $\int_{-1}^1 f(x)\,dx = \int_{-1}^0 (x+1)\,dx + \int_0^1 \cos(\pi x)\,dx = \frac{1}{2}(x+1)^2\Big|_{-1}^0 + \frac{1}{\pi}\sin(\pi x)\Big|_0^1$

$$= \frac{1}{2} + \frac{1}{\pi}\left(\sin\pi - \sin 0\right) = \frac{1}{2}$$

42. D $\Delta x = \frac{1}{3}$; $T = \frac{1}{2} \cdot \frac{1}{3}\left(1^2 + 2\left(\frac{4}{3}\right)^2 + 2\left(\frac{5}{3}\right)^2 + 2^2\right) = \frac{127}{54}$

43. E Solve $\frac{x}{2} = -1$ and $\frac{x}{2} = 2$; $x = -2,\ 4$

44. B Use the linearization of $f(x) = \sqrt[4]{x}$ at $x = 16$. $f'(x) = \frac{1}{4}x^{-\frac{3}{4}}$, $f'(16) = \frac{1}{32}$

$L(x) = 2 + \frac{1}{32}(x - 16)$; $f(16 + h) \approx L(16 + h) = 2 + \frac{h}{32}$

45. C This uses the definition of continuity of f at $x = x_0$.